Four Digits

A, B, C, and D are four distinct digits,

where $4 \times ABCD = DCBA$

What are the values of A, B, C, and D?

Solution to Four Digits

For clarity of explanation, label the digits in the right-hand side of the equation with apostrophes:

$$4 \times ABCD = D'C'B'A'$$

where $A = A'$, $B = B'$, etc.

When any number is multiplied by 4, the product is an even number, so A' must be even. And since the product D'C'B'A' has only four digits, we know that $4 \times A$ must be less than 10. From these two conditions, we can infer that **A = 2**. (It cannot be zero because $A' = 0$ implies that $D = 5 = D'$, so the right-hand side would be a number bigger than 5,000. But the left-hand side would be 4 times a 3-digit number no bigger than 987, so the left-hand side could not be bigger than 3,948.)

The product $4 \times D$ must end in the digit 2 ($= A'$). So D must be either 3 or 8. But D' must be at least 8 because the right-hand side of the equation must be bigger than 8,000 since the left-hand side is 4 times a number bigger than 2,000. So **D = 8**.

We can now see that $4 \times B$ must be less than 10, otherwise D' would not be equal to 8. So B is equal to either 0 or 1. For B' to be equal to 0, the sum of 3 (from the first digit of $4 \times D = 32$) and the product $4 \times C$ would need to end in 0. That is, $4 \times C$ would need to end in 7, which is impossible. So, we can conclude that **B = 1**.

So now the equation can be written: $4 \times 21C8 = 8C'12$

The product $4 \times C$ must end in the digit 8 , otherwise the right-hand side would not end in 12. So C must be either 2 or 7. It cannot be 2 because $A = 2$ and the digits are unique. So **C = 7**.

The equation is: $4 \times 2178 = 8712$.